AVIRAL CLASSES
CREATING SCHOLARS
JEE (ADVANCED), PMT & FOUNDATIONS

WEEKLY TEST TARGET- JEE MATHEAMTICS SOLUTION 10 NOVE 2019

51. (c) $m = \tan 60° = \sqrt{3}$. Therefore, equation of tangent is $y = \sqrt{3}x \pm \sqrt{1+3\cdot 16} \Rightarrow y = \sqrt{3}x \pm 7$.

52. (c) $E \equiv 4 + 9(3)^2 - 16(1) - 54(3) + 61 < 0$

Therefore, the point is inside the ellipse.

$\frac{4(x-2)^2}{36} + \frac{9(y-3)^2}{36} = 1$

Equation of major axis is $y - 3 = 0$ and point (1, 3) lies on it.

53. (a) $y = \frac{-l}{m}x + \frac{n}{m}$ is tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

$\frac{n}{m} = \pm\sqrt{b^2 + a^2\left(\frac{l}{m}\right)^2}$ or $n^2 = m^2b^2 + l^2a^2$.

54. (c) It is a fundamental concept.

55. (d) The point does not lie on ellipse .

56. (c) $SS_1 = T^2$

$\tan\theta = 2\frac{\sqrt{h^2 - ab}}{a+b}, a = 9$, $b = -4$ and $h = -12$.

57. (a) The tangent will be $y - 3 = m(x-2) \Rightarrow y - mx = 3 - 2m$.

But it is tangent to the given ellipse, therefore $m = 0, -1$. Hence tangents are $y = 3$ and $x + y = 5$.

58. (b) The tangent at $(a\cos\theta, b\sin\theta)$ to the ellipse is

$\frac{(a\cos\theta)x}{a^2} + \frac{(b\sin\theta)y}{b^2} = 1$ or $\frac{x}{(a/\cos\theta)} + \frac{y}{(b/\sin\theta)} = 1$

$\therefore$ Intercepts are, $h = \frac{a}{\cos\theta}$, $k = \frac{b}{\sin\theta} \Rightarrow \frac{a^2}{h^2} + \frac{b^2}{k^2} = 1$.

59. (a) Since, here *a* and *b* are interchanged.

60. (b) The normal at $P(a\cos\theta, b\sin\theta)$ is $ax\sec\theta - by\ \mathrm{cosec}\,\theta = a^2 - b^2$, where $a^2 = 14$, $b^2 = 5$

It meets the curve again at $Q(2\theta)$ *i.e.*, $(a\cos 2\theta, b\sin 2\theta)$.

$\therefore \frac{a}{\cos\theta}a\cos 2\theta - \frac{b}{\sin\theta}(b\sin 2\theta) = a^2 - b^2$

$\Rightarrow \frac{14}{\cos\theta}\cos 2\theta - \frac{5}{\sin\theta}(\sin 2\theta) = 14 - 5$

$\Rightarrow 18\cos^2\theta - 9\cos\theta - 14 = 0$

$\Rightarrow (6\cos\theta - 7)(3\cos\theta + 2) = 0 \Rightarrow \cos\theta = -\frac{2}{3}$.

61. (c) As we know that the line $lx + my + n = 0$ is normal to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$. But in this condition, we have to replace *l* by *m*, *m* by -1 and *n* by *c*, then the required condition is $c = \pm\frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}}$.

62. (d) For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, equation of normal at point (x_1, y_1),

$\Rightarrow \frac{(x - x_1)a^2}{x_1} = \frac{(y - y_1)b^2}{y_1}$; $\therefore (x_1, y_1) \equiv (0,3)$, $a^2 = 5$, $b^2 = 9$

$\Rightarrow \frac{(x-0)}{0}\ 5 = \frac{(y-3).9}{3}$ or $x = 0$ *i.e., y*-axis.

63. (b) $\frac{x-x_1}{x_1/a^2} = \frac{y-y_1}{y_1/b^2}$, which is the standard equation of normal at point (x_1, y_1).

In the given ellipse, $a^2 = 20, b^2 = \frac{180}{16}$.

Hence the equation of normal at the point $(2, 3)$ is

$\frac{x-2}{2/20} = \frac{y-3}{48/180} \Rightarrow 40(x-2) = 15(y-3)$

$\Rightarrow 8x - 3y = 7 \Rightarrow 3y - 8x + 7 = 0$.

64. (d) The equation of any normal to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $ax\ \sec\phi - by\ \text{cosec}\,\phi = a^2 - b^2$ .....(i)

The straight line $x\cos\alpha + y\sin\alpha = p$ will be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

If (i) and $x\cos\alpha + y\sin\alpha = p$ represent the same line $\frac{a\sec\phi}{\cos\alpha} = \frac{-b\,\text{cosec}\phi}{\sin\alpha} = \frac{a^2-b^2}{p}$

$\Rightarrow \cos\phi = \frac{ap}{(a^2-b^2)\cos\alpha}$, $\sin\phi = \frac{-bp}{(a^2-b^2)\sin\alpha}$

$\because \sin^2\phi + \cos^2\phi = 1$

$\Rightarrow \frac{b^2p^2}{(a^2-b^2)^2\sin^2\alpha} + \frac{a^2p^2}{(a^2-b^2)^2\cos^2\alpha} = 1$

$\Rightarrow p^2(b^2\text{cosec}^2\alpha + a^2\sec^2\alpha) = (a^2-b^2)^2$.

65. (b) The equation of any normal to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$ax\sec\theta - by\ \text{cosec}\theta = a^2 - b^2$ .....(i)

The straight line $lx + my + n = 0$ .....(ii)

will be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

If (i) and (ii) represent the same line

then, $\frac{a\sec\theta}{l} = \frac{b\,\text{cosec}\theta}{-m} = \frac{a^2-b^2}{-n}$

$\Rightarrow \cos\theta = \frac{-an}{l(a^2-b^2)}$ and $\sin\theta = \frac{bn}{m(a^2-b^2)}$

$\because \cos^2\theta + \sin^2\theta = 1$

$\therefore \frac{a^2n^2}{l^2(a^2-b^2)^2} + \frac{b^2n^2}{m(a^2-b^2)^2} = 1 \Rightarrow \frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$.

66. (c) Hyperbola is $\frac{x^2}{9} - \frac{y^2}{5} = 1$.

Hence point of contact is $\left[\frac{-9(1)}{\sqrt{9-5}}, \frac{-5}{\sqrt{9-5}}\right] \equiv \left[\frac{-9}{2}, \frac{-5}{2}\right]$.

Trick : Since the point $\left(-\frac{9}{2}, -\frac{5}{2}\right)$ satisfies both the equations.

67. (c) If $y = 2x + \lambda$ is tangent to given hyperabola, then $\lambda = \pm\sqrt{a^2m^2 - b^2} = \pm\sqrt{(100)(4) - 144} = \pm\sqrt{256} = \pm 16$.

68. (a) Tangent to $\frac{x^2}{1} - \frac{y^2}{3} = 1$ and perpendicular to $x + 3y - 2 = 0$ is given by $y = 3x \pm \sqrt{9-3} = 3x \pm \sqrt{6}$.

69. (b) Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Any tangent to hyperbola are $y = mx \pm \sqrt{a^2m^2 - b^2}$

Also tangent perpendicular to this is $y = \frac{-1}{m}x \pm \sqrt{\frac{a^2}{m^2} - b^2}$

Eliminating m, we get $x^2+y^2=a^2-b^2$.

70. (b) The tangent at (h,k) is $\frac{x}{4/h}-\frac{y}{3/k}=1$

$\therefore \frac{4}{h}=\frac{3}{k}\Rightarrow\frac{h}{k}=\frac{4}{3}$ .....(i)

and $3h^2-4k^2=12$ ....(ii)

As point (h,k) lies on it, using (i) and (ii), we get the tangent as $y-x=\pm1$.

71. $\frac{x^2}{(48/3)}+\frac{y^2}{(48/4)}=1$

$a^2=16, b^2=12 \Rightarrow e=\sqrt{1-\frac{b^2}{a^2}}=\frac{1}{2}$

Distance is $2ae=2\cdot4\cdot\frac{1}{2}=4$.

72. (a) The hyperbola is $\frac{x^2}{16}-\frac{y^2}{9}=1$. We have difference of focal distance $=2a=8$.

73. $e=\sqrt{1+\frac{b^2}{a^2}} \Rightarrow e^2=\frac{a^2+b^2}{a^2}$

$e_1=\sqrt{1+\frac{a^2}{b^2}} \Rightarrow e_1^2=\frac{b^2+a^2}{b^2} \Rightarrow \frac{1}{e_1^2}+\frac{1}{e^2}=1$.

74. $\frac{x^2}{3^2}-\frac{y^2}{4^2}=1$. Therefore $PS_1 \sim PS_2=2(3)=6$

75. $2a=8, 2b=6$

Difference of focal distances of any point of the hyperbola $=2a=8$.